THEORY OF SUBHARMONIC SYNCHRONIZATION OF NONLINEAR OSCILLATORS

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ABSTRACT

Synchronization is an important technique to provide frequency coherency of remotely located independent oscillators to the same frequency reference. To extend this technique to the millimeter wave frequencies of interest, subharmonic injection locking is used as a viable technique. This attractive method primarily relies on nonlinear characteristics of microwave devices, such as FET, to extend injection locking of millimeter wave oscillators to large subharmonic numbers. Important figure of merit of injection locked oscillators is locking range, and goal of this paper is to present analytical method to express locking range of the subharmonically locked oscillators in terms of nonlinear current voltage relationship. Experimental results of a subharmonically injection locked FET oscillator at 18GHz are also presented.

INTRODUCTION

Large aperture phased array antennas composed of many remotely located active T/R modules are designed for more advanced communication and imaging systems. For coherent operation of the phased array antennas, the active modules should be phase and frequency synchronized. Various techniques of frequency synchronization have been investigated and the most promising approach is injection locking of local oscillators, existing in each array or subarray modules, to the same injected frequency reference. The injection locking process can be accomplished at the frequencies in the proximity of the free-running oscillation frequency of the local oscillator, related to the frequency reference either by the fundamental, superharmonic, subharmonic, or a mixed frequency. All these techniques of injection locking rely on the nonlinear characteristics of active devices used in oscillators to force oscillation at the injected signal frequency. In general the topic of frequency control of oscillators falls under frequency entrainment or forced oscillations.

Forced oscillations in nonlinear oscillators, on a purely mathematical basis was first introduced by Van der Pol in 1927 [1], and has extensively been studied in standard text books [2, 3]. Equally interesting problem of parametric resonance, has also been analyzed on the basis of Van der Pol theory [4]. Van der Pol's mathematical approach is complex and difficult to implement for the microwave injection locked oscillators. However, the approach presented by Adler [5] and Jelonik [6], defines the frequency entrainment phenomena in terms of a first order nonlinear differential equation, which yields a figure of merit known as *locking range*. This figure of merit explains the performance of different types of oscillators, in a well defined system, by a common relationship relating the locking gain to the locking range by external Q of the oscillator. Although Adler's initial analysis assumed small-signal conditions, but his results have also been extended to large forced oscillations [7]. Adler's approach has been also thoroughly studied using nonlinear equivalent circuit model [8], and the describing function method [9].

An approach similar to Adler's figure of merit was extended to describe the super-harmonic locking range [10]; however, no theory is reported on the subharmonic locking range. The goal of this paper is to calculate the locking range for the subharmonically injection locked oscillators in an approach analogous to Adler's.

THEORY

Block diagram of a nonlinear oscillator under forced oscillation is shown in Fig. 1, which is governed by the following set of equations:

$$u = f(e) \tag{1}$$

$$H(D)u + v = e \tag{2}$$

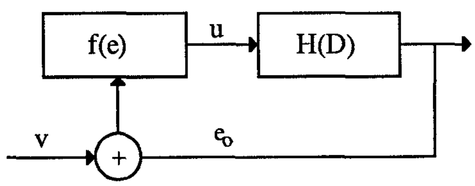

Fig. 1. Block diagram of a nonlinear oscillator.

where e and u denote the input and output respectively, f(e) is a nonlinear function describing the dependence of output on input, H is a linear function of differential operators D, and v is the synchronizing reference signal. For example in the simple case of FET, e is the input voltage across gate to source whereas u corresponds to the output drain current. The function f for this example is a complex function, if nonlinear transconductance as well as nonlinear Schottky contact capacitance are included in the device model. The present analysis assumes a real representation of function f rahter than a complex one (i.e., only nonlinear

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conductance), however a similar approach can be pursued for the general case of complex representation of function f.

Following Adler [5], the steady state solution is investigated. The linear part of H is approximated by a single-tuned resonant circuit with the quality factor Q. Let the transfer function of the linear part be expressed by [5, 7]:

$$H = \frac{H_o}{1+2iQ\Delta\omega/\omega} \qquad (3)$$

where $\Delta\omega$ is the frequency deviation from the resonant frequency of the single-tuned resonant circuit, ω. For small synchronizing signals, a steady state solutions is:

$$H(D)u_o = e_o = E\cos\omega t,$$

where E is the maximum amplitude of input signal. Substituting $e = e_o+v$ into Eq. (1) as the input driving force and expanding u in a Taylor series results in:

$$u = f(e_o + v) = f(e_o) + f^{(1)}(e_o)v + f^{(2)}(e_o)(v^2/2!) + \ldots = \sum_{m=0}^{\infty} f^{(m)}(e_o)(v^m/m!)$$

Let the subharmonic synchronizing signal be in the form of:

$$v = (V/2)\left\{e^{i(\omega_o t+\varphi)} + e^{-i(\omega_o t+\varphi)}\right\} = V\cos(\omega_o t + \varphi) \qquad (4)$$

where V and φ are maximum amplitude and phase of the subharmonic injection signal at frequency $\omega_o \approx \omega/s$, and s is an integer, signifying the order of the subharmonic number (s:1). The expansion of $f^{(m)}$ $(E\cos\omega t)$ in terms of the complete orthogonal basis set of $\exp(in\omega_o t)$ results in:

$$u = f(e_o + v) = \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{\infty} A_{m,n}(E)\, e^{in\omega_o t}(v^m/m!) \qquad (5)$$

where

$$A_{m,n}(E) = A_{-m,n}(E) = (1/2\pi)\int_{-\pi}^{+\pi} f^{(m)}(E\cos\alpha)\cos(n\alpha)d\alpha. \quad (6)$$

Simple relations based on trigonometric identities of $\cos(n\pm s)\alpha$ exist between the coefficients $A_{m,n}(E)$ expressed by Eq. (6):

$$[A_{m,(n-s)}(E) - A_{m,(n+s)}(E)]/2 = [n/(sE)]\, A_{(m-1),n} \qquad (7)$$

$$[A_{m,(n-s)}(E) + A_{m,(n+s)}(E)]/2 = d\, A_{(m-1),n}(E)/dE. \qquad (8)$$

Substituting Eq. (4) in Eq. (5), and use of relationships expressed by Eqs. (7) and (8) in addition to the harmonic balance method [11], Eq. (5) can be expressed as:

$$(u/2)\, e^{i\omega t} = e^{is\omega_o t}\Big\{A_{o,s} + V/(sE)\,[A_{o,1}\cos\varphi + (V/2)A_{1,2}\cos 2\varphi + (V/2)^2 A_{2,3}\cos 3\varphi + (V/2)^3 A_{3,4}\cos 4\varphi + (V/2)^4 A_{4,5}\cos 5\varphi + \ldots + (V/2)^p A_{p,p+1}\cos(p+1)\varphi + \ldots] + iV\,[dA_{0,1}/dE \sin\varphi + (V/2)\, dA_{1,2}/dE \sin 2\varphi + (V/2)^2\, dA_{2,3}/dE \sin 3\varphi + (V/2)^3\, dA_{3,4}/dE \sin 4\varphi + (V/2)^4\, dA_{4,5}/dE \sin 5\varphi + \ldots + (V/2)^p\, dA_{p,p+1}/dE \sin(p+1)\varphi + \ldots]\Big\}\, e^{is\omega_o t}$$

Replacing this expression in Eq. (2) in conjunction with Eq. (3), and separating real and imaginary parts, the following set of equations are derived:

$$E = H_o\Big\{A_{o,s} + V/(sE)\,[A_{o,1}\cos\varphi + (V/2)A_{1,2}\cos 2\varphi + (V/2)^2 A_{2,3}\cos 3\varphi + (V/2)^3 A_{3,4}\cos 4\varphi + (V/2)^4 A_{4,5}\cos 5\varphi + \text{higher order terms}]\Big\}$$

$$E(2\Delta\omega/\omega)Q = V H_o\, d/dE\,\Big\{A_{o,1}(E)\sin\varphi + (V/2)A_{1,2}(E)\cos 2\varphi + (V/2)^2 A_{2,3}(E)\cos 3\varphi + (V/2)^3 A_{3,4}(E)\cos 4\varphi + (V/2)^4 A_{4,5}(E)\cos 5\varphi + \text{higher order terms}\Big\}.$$

In the above separation of real and imaginary parts, function f(e) is assumed to be a real function. This is a valid assumption for devices with predominantly nonlinear conductance. However in general, if function f(e) is complex, mathematics is straight forward and a similar approach as presented can be pursued. The above expression can be solved to calculate locking range of a subharmonically injection locked oscillator at subharmonic frequency of $\omega \approx \omega_o s$:

$$\Delta\omega_s = [(\omega/2Q)(V/E)]\, H_o\, d/dE\, \{A_{o,1}(E)\sin\varphi + (V/2)A_{1,2}(E)\cos 2\varphi + (V/2)^2 A_{2,3}(E)\cos 3\varphi + (V/2)^3 A_{3,4}(E)\cos 4\varphi + (V/2)^4 A_{4,5}(E)\cos 5\varphi + \text{higher order terms}\} \quad (9)$$

The first square bracket term is identified as the Adler's figure of merit for locking range at the fundamental frequency. This term can be presented in terms of injected and output power as:

$$2\Delta\omega_A = (\omega/Q)(V/E) = (\omega/Q)\sqrt{(P_i/P_o)}\,,$$

where P_i and P_o are the injected power and the output power of the injection locked oscillator. At both ends of locking range, the phase of difference between the sth harmonic of the injected signal and the slave oscillator output is $\pm\pi/2$, therefore a phase shift of $\varphi = \pm(\pi/2s)$ corresponds to the edges of the locking range.

ANALYSIS

Input-output nonlinear characteristics of active nonlinear microwave devices, such as FET, can be represented in terms of a polynominal relating current-voltage as:

$$i = f(e) = \sum_{n=1}^{\infty} a_n e^n = -a_1 e + a_2 e^2 + a_3 e^3 + a_4 e^4 + a_5 e^5 + \ldots$$

where a_n in general is a complex quantity relating the level dependent nonlinear conductance and capacitance of active devices and a_1 is complex number with a positive real part so that source of power generation can be accounted for. In the present analysis only nonlinear conductance is considered, hence only real numbers are selected for a_n. To calculate locking range of the subharmonically injection locked oscillators at subharmonic factor s, the input-output dependence presented by function f(e) is utilized in conjunction with Eq. (6).

More specifically, the expression predicting locking range for subharmonic factor s=2 is derived as:

$$2\Delta\omega_s = 2\Delta\omega_A \frac{4V(a_2+3a_4E^2)}{a_5E^4+2a_3E^2-4a_1}$$

This expression is derieved by recognizing that the only nonzero terms in Eq. (9) are $A_{o,2}$ and $A_{1,2}$. Contribution of $A_{1,2}$ to H_o is negated due to the fact that the $\cos 2\varphi$ at end of locking range (i.e., $\varphi=\pi/4$) is zero, hence the overall contribution is limited to $A_{o,2}$. Therefore, only derivative of $A_{1,2}$ term contributes to the locking range expression.

In a similar fashion locking range dependence on the injected signal for s=3 can be derived using Eq. (6). This will results in a quadratic of on injected voltage as:

$$2\Delta\omega_s = 2\Delta\omega_A \frac{2V^2(3a_3+15a_5E^2)}{a_5E^4+2a_3E^2-4a_1}$$

where only contribution of $A_{o,3}$ and $A_{2,3}$ are necessary to include and others terms in Eq. (6) are either zero or their contribution is very negligible. Once again since $\cos 3\varphi$ will be zero at the edge of locking range for subharmonic factor s=3, only derivative of $A_{2,3}$ will contribute to the locking range.

On the other hand at subharmonic fasctor of s=4, the locking range can be simplified as:

$$2\Delta\omega_s = 2\Delta\omega_A \frac{12V^3a_4}{a_5E^4+2a_3E^2-4a_1}$$

where only $A_{o,4}$ and $A_{3,4}$ terms have nonzero contribution. Following this approach, a pattern becomes evident predicting the terms contributing to the locking range. In fact for subharmonic factor s only the $A_{o,s}$ and $A_{s-1,s}$ terms should be considered in the locking range expression. The above expressions clearly define dependence of locking range on the device nonlinearity, output power of oscillator as well as the injected power level.

EXPERIMENT

To validate results of the theoretical analysis, an oscillator was constructed using Avantek transitor. The free-running oscillation output power of 10dBm at 17.956GHz is measured for this oscillator. Nonlinear equivalent circuit model of this oscillator was determined by measuring load admittance for maximum output power using automated load-pull measurement available from David Sarnoff Research Center. Power level at harmonics of free-running oscillation frequency at the given load was also monitored on a spectrum analyzer, so that nonlinear current and voltage relationship can be identified. Based on the equivalent circuit model and extent of the nonlinear transconductance of the transitor locking range for subharmonic factors of 2, 3, and 4 were calculated. Experimental results of locking range as a function of injected power for the first three subharmonic factors are shown in Fig. 2. Detailed comparison of theoretical and experimental results will be presented at the meeting.

CONCLUSION

General expressions for theoretical prediction of subharmonic injection locking range of oscillators with nonlinear input-output relationship was presented. This analysis was extended to describe oscillators realized using real nonlinear input voltage and output current relationships. In particular analytical results for an oscillator presented by a fifth order polynominal describing nonlinear current and voltage relationship for subharmonic factors of 2, 3, and 4. Experiments relating subharmonic locking range of FET oscillator of 18GHz to the injected signal level were also reported.

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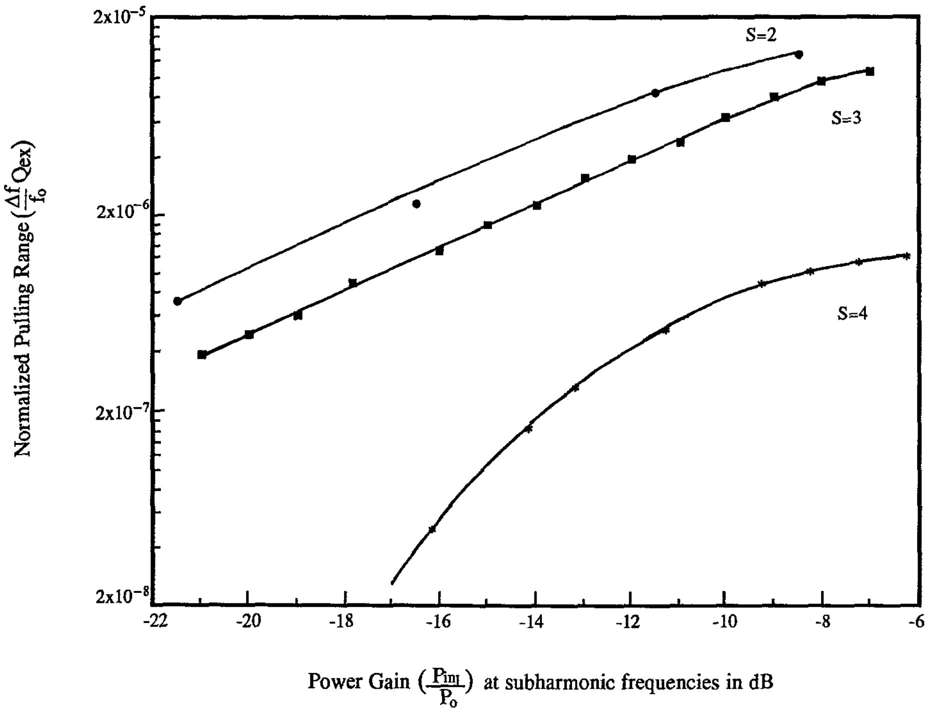

Fig. 2. Locking range of an injection locked FET oscillator with an output power of 10dBm at 18GHz as a function of various subharmonic numbers of s=2, s=3, and s=4.

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